Technical Report 1055

An Additional Metric for Communicating Group Performance Differences

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The common practice of expressing group performance differences in standard deviation units conveys useful but limited information. Reporting the percentage of the time a member from a lower-performing group is expected to outperform a member from a higher performing group would enhance understanding of the magnitude of the difference. Furthermore, the proposed percentage metric is able to easily deal with group variability differences in addition to mean group differences. An analytical approach was used to convert group performance differences from standard deviation units to the proposed metric. The slope of the relationship between the two metrics is nearly linear through a one standard deviation group performance difference. Tables are presented that can be used to convert group performance differences in standard deviation units to the new metric.

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An Additional Metric for Communicating Group Performance Differences

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FOREWORD

Researchers are often called upon to communicate research findings to decision-makers in relevant and meaningful ways. One area of difficulty has been to convey the meaningfulness and magnitude of group differences. The metric of choice to communicate group differences has always been the standard deviation. The standard deviation metric is one which is well known and understood by researchers but difficult for those not familiar with statistics. Even for researchers thoroughly familiar with the metric, it still fails to convey easily meaningful group difference information.

In response to the difficulty of conveying meaningful magnitude information regarding group differences with the standard deviation metric, an additional metric was conceived which could be easily understood by researchers and decision-makers alike and would not distort the information provided by the standard deviation metric. Its primary advantage would be that the magnitude information would be self-evident whereas the standard deviation's magnitude information is not. Such a metric was developed with these properties and is reported herein. It has been used by ARI researchers to convey group difference information to military decision-makers and it has been found to be easily understood by decision-makers and very well received.

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I gratefully thank Brian Davis of the U. S. Army Training Support Center who wanted a way to communicate group differences in a way which decision-makers could understand. His encouragement to develop an alternative method was the seed for this work.

AN ADDITIONAL METRIC FOR COMMUNICATING GROUP PERFORMANCE DIFFERENCES

EXECUTIVE SUMMARY

Research Requirement:

Develop a metric to convey group difference information (i.e., direction and magnitude) which could be easily communicated and understood by decision-makers and researchers. This metric would convey the information in such a way that the magnitude of group differences would be self-evident. In addition, the new metric would convey this information with little or no distortion in the range where most group differences occur.

Procedure:

A metric was conceived, its formula developed, and a solution for the formula was achieved using numerical methods. A direct evaluation of the formula was not possible since it does not have a closed form. In addition to creating the new metric for the case when the two groups possessed equal variability, a solution was also developed for the cases where the variability of the groups differed.

Findings:

The proposed metric is nearly linear below a group difference of 1 standard deviation. Since most observed group differences fall below 1 standard deviation, this indicates that most of the magnitude information carried by the standard deviation difference metric is also contained in the new metric. This characteristic of the new metric, its simplicity, its directness, and its ability to address variability as well as mean differences makes it well suited to convey group performance difference information.

Utilization of Findings:

The new metric can and has been used to convey group difference information to other decision-makers and other researchers at conferences. It has been very well received and easily understood. The tables provided make the conversion from the standard deviation metric to the new metric simple and accessible.

AN ADDITIONAL METRIC FOR COMMUNICATING GROUP PERFORMANCE DIFFERENCES

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AN ADDITIONAL METRIC FOR COMMUNICATING GROUP PERFORMANCE DIFFERENCES

INTRODUCTION

Representing group performance differences in the standard deviation metric is appropriate and commonly done in studies which examine inter-group performance differences (Coleman et al, 1966; Grant & Bray, 1970; Hunter, 1983; Hunter, Schmidt, & Rauschenberger, 1977; Hyde, Fennema, & Lamon, 1990; Hyde & Linn, 1988). The standard deviation metric informs on the magnitude of the group performance difference while maintaining an interval scale.

Yet, even with all its positive features, including the ability to perform inferential statistical analysis on it, the standard deviation metric lacks directness. Psychometricians routinely place examinee scores on a percentile metric to more easily convey how an examinee performed compared to others. An examinee with a percentile score of 95%, for example, understands that she performed better than 95% of those who took the test. Telling the same examinee that his/her score was 1.648 (i.e., her z-score) conveys little information. If they want to understand how they did relative to others they have to know the shape of the score distribution and how to convert their z-score to a percentile score.

This logic can be applied to the group as well as to the individual. Knowing that one group performs 0.50 standard deviations lower than another group conveys minimal information. Most researchers understand that 0.50 standard deviations is a sizeable difference whereas a 0.10 standard deviation difference is not. Most researchers, however, do not know how this might translate to a percent metric. For example, what percent of the time will someone from a lower-performing group outperform someone from a higher-performing group?

Thurstone (1927) developed the law of comparative judgment to be able to compare objects, responses, and individuals at a more practical level. A good example to which the law of comparative judgment can be applied was borrowed from Nunnally (1978) and involves two runners. One runner is on average faster by a margin of $\underline{D}$ (a standardized mean difference score). Although $\underline{D}$ provides a good summary measure of how these individuals perform relative to each other it does not indicate how likely it is that on any given day, the slower runner will beat the faster runner. If, for example, their running times were normally distributed and indicated a 0.50 standard deviation time advantage in favor of the faster runner, then how often would the slower runner beat the faster runner? The answer is the slower runner would beat the faster runner in 36% of the races. The standard deviation difference together with the percent metric gives a much clearer view of the difference between the runners than the standard deviation difference alone.

The law of comparative judgment and the runner analogy can be extended to group differences. Each runner becomes a group and each running time becomes an individual within that group. Running times become performance on a test or an aptitude level. The question then is: When an individual from each group is randomly paired with an individual from the other group, what is the probability that the individual from one of the groups will outperform (i.e., outrace or beat in terms of the analogy) the individual from the other group? This is equivalent to asking what percent of the time will someone from a lower-performing group outperform someone from a higher-performing group?

Specifically, it is proposed that the standard deviation metric be converted to the percent of the time an individual from the lower-performing group performs better than someone from a higher-performing group. When the group performance difference between two groups is zero (and the two groups are distributed the same in terms of shape and dispersion) it would indicate that members of each group underperform and outperform each other equally often (i.e., 50%). Note that when the group performance difference is zero the labels "lower-performing group" and "higher-performing group" are inappropriate. As one group's performance, on average, decreases relative to the other, then the percent of the time a lower-performing group member outperforms a higher-performing one drops below 50%.

Easily Accommodates Group Variability Differences

Feingold (1995) presented very clearly why group variability differences can affect the relative proportion of individuals at the tail ends of group distributions (i.e., tail ratios). Specifically, he demonstrated that lack of mean differences in the presence of group variability differences can affect the tail ratios substantially. The mean group difference is not the whole story, the variability must also be considered when quantifying group differences. Feingold (1995) and Hedges and Friedman (1993) suggest that the mean difference be adjusted for the variability difference. While this is an acceptable approach, the adjusted standardized mean difference becomes more and more detached from a direct interpretation. In contrast, the percentage metric proposed can easily accommodate inter-group mean and variability differences and the interpretation of the metric remains the same: percent of the time someone from a lower-performing group will outperform someone from a higher-performing group.

METHOD

In order to solve for the proportion of the time that a randomly selected lower-performing group member is expected to outperform a randomly-selected higher-performing group member given inter-group mean and variability differences, we let $f_1(x)$ and $f_2(x)$ represent two normal density functions for group 1 (higher-performing group) and group 2 (lower-performing group), respectively. The proportion of the time that the randomly selected lower-performing group member performs at or above performance level x is represented by:

$$F_2(x) = \int_x^{\infty} f_2(x)\ dx. \tag{1}$$

Then, in order to calculate the proportion of the time when the randomly selected lower-performing group member is expected to outperform the randomly selected higher-performing group member, the following formula is used:

$$p = \int_{-\infty}^{+\infty} f_1(x)\ F_2(x)\ dx\ . \qquad \textbf{(2)}$$

Equation (2) multiplies the probability of selecting someone from the higher-performing group with a score of x with the probability of selecting someone from the lower-performing group who will outscore the higher-performing group member selected. This is done for all possible values of x and all such products are summed.

Since equation (2) does not have a closed form, numerical methods were used to evaluate p. The mean performance difference between the two groups (i.e., delta) was varied from 0 to 2.99 standard deviations in intervals of 0.01, and the ratio of the standard deviation between the two groups was varied from 0.6 to 1.4 in intervals of 0.2.

For each delta level and standard deviation ratio combination the higher-performing group's normal distribution was divided into 48,000 intervals in the standard score range from -6 to +6. Each interval thus spanned 0.00025 standard deviation units of the higher-performing group's distribution. For each of the 48,000 intervals, the area of the higher-performing group's distribution covered by the interval was multiplied by the area of the lower-performing group's normal distribution falling above the standard score where the higher-performing group's interval begins.

The point (i.e., z) at which the lower-performing group's interval begins is determined by delta and the ratio of the standard deviation of the two groups (i.e., the standard deviation of the higher-performing group divided by the standard deviation of the lower-performing group). It is further determined as a function of the higher-performing group which assumes a mean of zero and a standard deviation of one. Thus, to obtain the z value appropriate to the lower-performing distribution, the z value from a standard normal distribution for the higher-performing group is multiplied by the ratio of the higher-performing group standard deviation over the lower-performing group standard deviation, and the absolute value of delta is added to it.

The sum of the 48,000 products accurately yields the proportion of the time a lower-performing group member is expected to outperform a higher-performing group member. Monte Carlo analysis at various group performance difference values revealed that dividing the normal curve into 48,000 intervals yielded accuracy to the fourth decimal place. The index p has a maximum value of 0.50 and a minimum value of 0. Multiplying p by 100 converts it from a proportion to a percentage (P).

The new index, P, is a percent and has a range of zero to 50 when delta is zero or greater. An index value of 50 indicates that members of each group are expected to outperform each other equally often and occurs when the mean performance difference is zero. A P value of zero occurs when the performance distributions for the two groups have no overlap . This is theoretically impossible with normal distributions but for practical purposes it is possible to obtain a value so close to zero that stating a minimum of zero is accurate.

RESULTS

Table 1 presents the percent of the time one expects a randomly selected lower-performing group member to outperform a randomly selected higher-performing group member, as a function of the standard deviation difference in the mean scores of the two groups (i.e., delta) when the ratio of the standard deviations of the two groups is one. This percentage is 50 when delta is zero. That is, when the two groups do not differ in mean performance, one expects that one group's randomly selected member would outperform the other group's randomly selected member 50% of the time. Increasing the average group difference to 0.50 standard deviations lowers this percentage to 36.18. This means that when delta is 0.50, a lower-performing group member is expected to outperform a higher-performing group member only 36.18% of the time, and, conversely, a higher-performing group member is expected to outperform a lower-performing group member 63.82% (i.e., 100-36.18) of the time. At a delta of 1.00, the lower-performing group member outperforms the higher-performing group member only 23.97% of the time, and when delta reaches 2.00 this percentage drops to 7.86%.

Tables 2 through 5 present the percent of the time one expects a lower-performing group member to outperform a higher-performing group member, as a function of delta when the ratio of the standard deviations of the two groups is not one. This percentage is 50 when delta is zero regardless of the ratio of the standard deviations. In addition, when the lower-performing group's standard deviation is smaller relative to the higher-performing group's, then the percentage for any given value of delta is larger relative to when the standard deviation ratio is one. The converse is true when the higher-performing group's standard deviation is smaller. Finally, the difference in the percentages at different standard deviation ratios are larger for larger deltas (i.e., up to a delta of about 1.30 to 1.50 depending on the standard deviation ratio) and the more the standard deviation ratios differ. For example, the maximum difference between standard deviation ratios of 0.6 and 1.4 occurs at a delta of 1.41. At a delta of 1.41 and a standard deviation ratio of 0.6 the P index has a value of 11.33 while at a standard deviation ratio of 1.4 the P value is 20.63, or 9.30 points higher.

Figure 1 depicts the relationship between P and standard deviation metrics for various standard deviation ratios. For all standard deviation ratios the function was negatively decelerated across increasing delta values but was essentially linear for values below a delta of 0.70 and nearly linear up to a delta of 1.00. The deviation from linearity progressively accelerated beyond a delta value of 1.00 and became quite pronounced at the highest delta values examined. The near linearity of the proposed percentage metric, below a 1.00 standard deviation group performance difference, indicates that a one-to-one magnitude correspondence exists between the standard deviation metric and the proposed percentage metric in this interval at all standard deviation ratios.

DISCUSSION

The proposed P metric is nearly linear below a delta of 1.00. Since most observed delta values fall below 1.00, this indicates that most of the magnitude information carried by the standard deviation difference metric is also contained in the proposed percent metric. This characteristic of the proposed metric, its simplicity, its directness, and its ability to address variability as well as mean differences makes it well suited to convey group performance difference information. The tables provided makes the conversion from the standard deviation metric to the percent metric simple and accessible. The P metric would not substitute the tail ratio described by Hedges and Friedman (1993) and Feingold (1995) but would rather provide a global view of group difference across the entire score range. The tail ratio would still be useful to examine group differences in specific score ranges.

Finally, although the P index has some definitional overlap with Kendall's tau, the two indices are in fact different. If two persons are drawn randomly from an applicant pool, the difference between the probability that they will have the same order on X and Y and the probability that they will have different orders on X and Y is equal to tau. Although it may appear from this definition that tau is similar to the P index, it is not for two reasons: 1) the two persons selected could both be from the same group (e.g., lower-performing group) whereas the P index purposely selects one person from each group, and 2) tau is not directional with respect to the order on X and Y whereas the P index considers the direction of the order difference on X and Y.

Table 1

Percent of the Time a Randomly Selected Lower-Performing Group Member is Expected to Outperform a Randomly Selected Higher-Performing Group Member as a Function of Inter-Group Mean Performance Difference When the Groups Have Equal Standard Deviations

Delta	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.00	50.00	49.72	49.44	49.16	48.88	48.59	48.31	48.03	47.75	47.47
0.10	47.18	46.90	46.62	46.34	46.06	45.78	45.50	45.22	44.94	44.66
0.20	44.38	44.10	43.82	43.54	43.27	42.99	42.71	42.43	42.16	41.88
0.30	41.60	41.33	41.05	40.78	40.50	40.23	39.96	39.68	39.41	39.14
0.40	38.87	38.60	38.33	38.06	37.79	37.52	37.25	36.98	36.72	36.45
0.50	36.19	35.92	35.66	35.39	35.13	34.87	34.61	34.35	34.09	33.83
0.60	33.57	33.31	33.06	32.80	32.55	32.29	32.04	31.79	31.53	31.28
0.70	31.03	30.78	30.54	30.29	30.04	29.80	29.55	29.31	29.07	28.82
0.80	28.58	28.34	28.10	27.87	27.63	27.39	27.16	26.92	26.69	26.46
0.90	26.23	26.00	25.77	25.54	25.32	25.09	24.87	24.64	24.42	24.20
1.00	23.98	23.76	23.54	23.32	23.11	22.89	22.68	22.47	22.26	22.05
1.10	21.84	21.63	21.42	21.22	21.01	20.81	20.61	20.41	20.21	20.01
1.20	19.81	19.61	19.42	19.22	19.03	18.84	18.65	18.46	18.27	18.09
1.30	17.90	17.72	17.53	17.35	17.17	16.99	16.81	16.64	16.46	16.29
1.40	16.11	15.94	15.77	15.60	15.43	15.26	15.10	14.93	14.77	14.61
1.50	14.44	14.28	14.13	13.97	13.81	13.66	13.50	13.35	13.20	13.05
1.60	12.90	12.75	12.60	12.46	12.31	12.17	12.03	11.88	11.74	11.61
1.70	11.47	11.33	11.20	11.06	10.93	10.80	10.67	10.54	10.41	10.28
1.80	10.16	10.03	9.91	9.78	9.66	9.54	9.42	9.31	9.19	9.07
1.90	8.96	8.84	8.73	8.62	8.51	8.40	8.29	8.18	8.08	7.97
2.00	7.87	7.76	7.66	7.56	7.46	7.36	7.26	7.16	7.07	6.97
2.10	6.88	6.79	6.69	6.60	6.51	6.42	6.33	6.25	6.16	6.08
2.20	5.99	5.91	5.82	5.74	5.66	5.58	5.50	5.42	5.35	5.27
2.30	5.19	5.12	5.05	4.97	4.90	4.83	4.76	4.69	4.62	4.55
2.40	4.49	4.42	4.35	4.29	4.22	4.16	4.10	4.04	3.98	3.92
2.50	3.86	3.80	3.74	3.68	3.63	3.57	3.51	3.46	3.41	3.35
2.60	3.30	3.25	3.20	3.15	3.10	3.05	3.00	2.95	2.90	2.86
2.70	2.81	2.77	2.72	2.68	2.63	2.59	2.55	2.51	2.47	2.43
2.80	2.39	2.35	2.31	2.27	2.23	2.19	2.16	2.12	2.09	2.05
2.90	2.02	1.98	1.95	1.91	1.88	1.85	1.82	1.79	1.76	1.73

Notes. Delta represents the inter-group mean performance difference in standard deviation units.

Table 3

Percent of the Time a Randomly Selected Lower-Performing Group Member is Expected to Outperform a Randomly Selected Higher-Performing Group Member as a Function of Inter-Group Mean Performance Difference When the Standard Deviation Ratio is 0.8[a]

Delta	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.00	50.00	49.69	49.38	49.07	48.76	48.45	48.13	47.82	47.51	47.20
0.10	46.89	46.58	46.27	45.96	45.65	45.34	45.03	44.72	44.41	44.11
0.20	43.80	43.49	43.18	42.88	42.57	42.26	41.96	41.65	41.35	41.05
0.30	40.74	40.44	40.14	39.84	39.53	39.23	38.93	38.64	38.34	38.04
0.40	37.74	37.45	37.15	36.86	36.56	36.27	35.98	35.68	35.39	35.10
0.50	34.81	34.53	34.24	33.95	33.67	33.38	33.10	32.82	32.53	32.25
0.60	31.97	31.69	31.42	31.14	30.87	30.59	30.32	30.05	29.77	29.50
0.70	29.24	28.97	28.70	28.44	28.17	27.91	27.65	27.39	27.13	26.87
0.80	26.61	26.36	26.10	25.85	25.60	25.35	25.10	24.85	24.60	24.36
0.90	24.11	23.87	23.63	23.39	23.15	22.91	22.68	22.44	22.21	21.98
1.00	21.75	21.52	21.29	21.06	20.84	20.62	20.39	20.17	19.95	19.74
1.10	19.52	19.31	19.09	18.88	18.67	18.46	18.25	18.05	17.84	17.64
1.20	17.44	17.24	17.04	16.84	16.65	16.45	16.26	16.07	15.88	15.69
1.30	15.50	15.32	15.13	14.95	14.77	14.59	14.41	14.24	14.06	13.89
1.40	13.72	13.55	13.38	13.21	13.04	12.88	12.71	12.55	12.39	12.23
1.50	12.08	11.92	11.76	11.61	11.46	11.31	11.16	11.01	10.87	10.72
1.60	10.58	10.44	10.29	10.16	10.02	9.88	9.75	9.61	9.48	9.35
1.70	9.22	9.09	8.96	8.84	8.71	8.59	8.47	8.35	8.23	8.11
1.80	7.99	7.88	7.76	7.65	7.54	7.43	7.32	7.21	7.11	7.00
1.90	6.90	6.79	6.69	6.59	6.49	6.39	6.30	6.20	6.10	6.01
2.00	5.92	5.83	5.74	5.65	5.56	5.47	5.39	5.30	5.22	5.13
2.10	5.05	4.97	4.89	4.81	4.74	4.66	4.58	4.51	4.44	4.36
2.20	4.29	4.22	4.15	4.08	4.01	3.95	3.88	3.82	3.75	3.69
2.30	3.63	3.56	3.50	3.44	3.38	3.33	3.27	3.21	3.16	3.10
2.40	3.05	2.99	2.94	2.89	2.84	2.79	2.74	2.69	2.64	2.59
2.50	2.55	2.50	2.46	2.41	2.37	2.32	2.28	2.24	2.20	2.16
2.60	2.12	2.08	2.04	2.00	1.96	1.93	1.89	1.85	1.82	1.78
2.70	1.75	1.72	1.68	1.65	1.62	1.59	1.56	1.53	1.50	1.47
2.80	1.44	1.41	1.38	1.36	1.33	1.30	1.28	1.25	1.23	1.20
2.90	1.18	1.15	1.13	1.11	1.08	1.06	1.04	1.02	1.00	0.98

Notes. Delta represents the inter-group mean performance difference in standard deviation units.
[a] Ratio computed as higher-performing group standard deviation divided by lower-performing group standard deviation.

Table 2

Percent of the Time a Randomly Selected Lower-Performing Group Member is Expected to Outperform a Randomly Selected Higher-Performing Group Member as a Function of Inter-Group Mean Performance Difference When the Standard Deviation Ratio is 0.6[a]

Delta	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.00	50.00	49.66	49.32	48.98	48.63	48.29	47.95	47.61	47.27	46.93
0.10	46.59	46.25	45.90	45.56	45.22	44.89	44.55	44.21	43.87	43.53
0.20	43.19	42.86	42.52	42.19	41.85	41.52	41.18	40.85	40.52	40.18
0.30	39.85	39.52	39.19	38.86	38.53	38.21	37.88	37.55	37.23	36.91
0.40	36.58	36.26	35.94	35.62	35.30	34.98	34.66	34.35	34.03	33.72
0.50	33.41	33.10	32.79	32.48	32.17	31.86	31.56	31.25	30.95	30.65
0.60	30.35	30.05	29.75	29.45	29.16	28.87	28.57	28.28	27.99	27.71
0.70	27.42	27.13	26.85	26.57	26.29	26.01	25.73	25.46	25.18	24.91
0.80	24.64	24.37	24.10	23.83	23.57	23.31	23.04	22.78	22.53	22.27
0.90	22.02	21.76	21.51	21.26	21.01	20.77	20.52	20.28	20.04	19.80
1.00	19.56	19.32	19.09	18.86	18.63	18.40	18.17	17.95	17.72	17.50
1.10	17.28	17.06	16.84	16.63	16.42	16.21	16.00	15.79	15.58	15.38
1.20	15.18	14.98	14.78	14.58	14.38	14.19	14.00	13.81	13.62	13.43
1.30	13.25	13.07	12.89	12.71	12.53	12.35	12.18	12.01	11.83	11.67
1.40	11.50	11.33	11.17	11.01	10.85	10.69	10.53	10.38	10.22	10.07
1.50	9.92	9.77	9.62	9.48	9.33	9.19	9.05	8.91	8.77	8.64
1.60	8.50	8.37	8.24	8.11	7.98	7.86	7.73	7.61	7.49	7.37
1.70	7.25	7.13	7.01	6.90	6.79	6.67	6.56	6.45	6.35	6.24
1.80	6.14	6.03	5.93	5.83	5.73	5.63	5.54	5.44	5.35	5.26
1.90	5.16	5.07	4.98	4.90	4.81	4.73	4.64	4.56	4.48	4.40
2.00	4.32	4.24	4.16	4.09	4.01	3.94	3.87	3.80	3.73	3.66
2.10	3.59	3.52	3.45	3.39	3.33	3.26	3.20	3.14	3.08	3.02
2.20	2.96	2.90	2.85	2.79	2.74	2.68	2.63	2.58	2.53	2.48
2.30	2.43	2.38	2.33	2.29	2.24	2.20	2.15	2.11	2.06	2.02
2.40	1.98	1.94	1.90	1.86	1.82	1.78	1.75	1.71	1.67	1.64
2.50	1.60	1.57	1.54	1.50	1.47	1.44	1.41	1.38	1.35	1.32
2.60	1.29	1.26	1.23	1.21	1.18	1.15	1.13	1.10	1.08	1.05
2.70	1.03	1.01	0.98	0.96	0.94	0.92	0.90	0.88	0.86	0.84
2.80	0.82	0.80	0.78	0.76	0.74	0.73	0.71	0.69	0.68	0.66
2.90	0.64	0.63	0.61	0.60	0.59	0.57	0.56	0.54	0.53	0.52

Notes. Delta represents the inter-group mean performance difference in standard deviation units.
[a] Ratio computed as higher-performing group standard deviation divided by lower-performing group standard deviation.

Table 4

Percent of the Time a Randomly Selected Lower-Performing Group Member is Expected to Outperform a Randomly Selected Higher-Performing Group Member as a Function of Inter-Group Mean Performance Difference When the Standard Deviation Ratio is 1.2[a]

Delta	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.00	50.00	49.75	49.49	49.24	48.98	48.73	48.47	48.22	47.96	47.71
0.10	47.45	47.20	46.94	46.69	46.43	46.18	45.92	45.67	45.42	45.16
0.20	44.91	44.66	44.40	44.15	43.90	43.65	43.39	43.14	42.89	42.64
0.30	42.39	42.14	41.89	41.64	41.39	41.14	40.89	40.64	40.39	40.15
0.40	39.90	39.65	39.40	39.16	38.91	38.67	38.42	38.18	37.93	37.69
0.50	37.45	37.21	36.96	36.72	36.48	36.24	36.00	35.76	35.52	35.29
0.60	35.05	34.81	34.58	34.34	34.10	33.87	33.64	33.40	33.17	32.94
0.70	32.71	32.48	32.25	32.02	31.79	31.56	31.33	31.11	30.88	30.66
0.80	30.43	30.21	29.98	29.76	29.54	29.32	29.10	28.88	28.66	28.45
0.90	28.23	28.01	27.80	27.58	27.37	27.16	26.94	26.73	26.52	26.31
1.00	26.11	25.90	25.69	25.49	25.28	25.08	24.87	24.67	24.47	24.27
1.10	24.07	23.87	23.67	23.47	23.28	23.08	22.89	22.70	22.50	22.31
1.20	22.12	21.93	21.74	21.55	21.37	21.18	21.00	20.81	20.63	20.45
1.30	20.27	20.09	19.91	19.73	19.55	19.38	19.20	19.03	18.85	18.68
1.40	18.51	18.34	18.17	18.00	17.83	17.67	17.50	17.34	17.17	17.01
1.50	16.85	16.69	16.53	16.37	16.21	16.06	15.90	15.74	15.59	15.44
1.60	15.29	15.14	14.99	14.84	14.69	14.54	14.40	14.25	14.11	13.97
1.70	13.82	13.68	13.54	13.41	13.27	13.13	12.99	12.86	12.73	12.59
1.80	12.46	12.33	12.20	12.07	11.94	11.82	11.69	11.56	11.44	11.32
1.90	11.19	11.07	10.95	10.83	10.71	10.60	10.48	10.36	10.25	10.14
2.00	10.02	9.91	9.80	9.69	9.58	9.47	9.36	9.26	9.15	9.05
2.10	8.94	8.84	8.74	8.64	8.54	8.44	8.34	8.24	8.14	8.05
2.20	7.95	7.86	7.76	7.67	7.58	7.49	7.40	7.31	7.22	7.13
2.30	7.05	6.96	6.88	6.79	6.71	6.62	6.54	6.46	6.38	6.30
2.40	6.22	6.14	6.07	5.99	5.91	5.84	5.77	5.69	5.62	5.55
2.50	5.48	5.41	5.34	5.27	5.20	5.13	5.06	5.00	4.93	4.87
2.60	4.80	4.74	4.68	4.61	4.55	4.49	4.43	4.37	4.31	4.25
2.70	4.20	4.14	4.08	4.03	3.97	3.92	3.86	3.81	3.76	3.70
2.80	3.65	3.60	3.55	3.50	3.45	3.40	3.36	3.31	3.26	3.22
2.90	3.17	3.12	3.08	3.04	2.99	2.95	2.91	2.86	2.82	2.78

Notes. Delta represents the inter-group mean performance difference in standard deviation units.
[a] Ratio computed as higher-performing group standard deviation divided by lower-performing group standard deviation.

Table 5

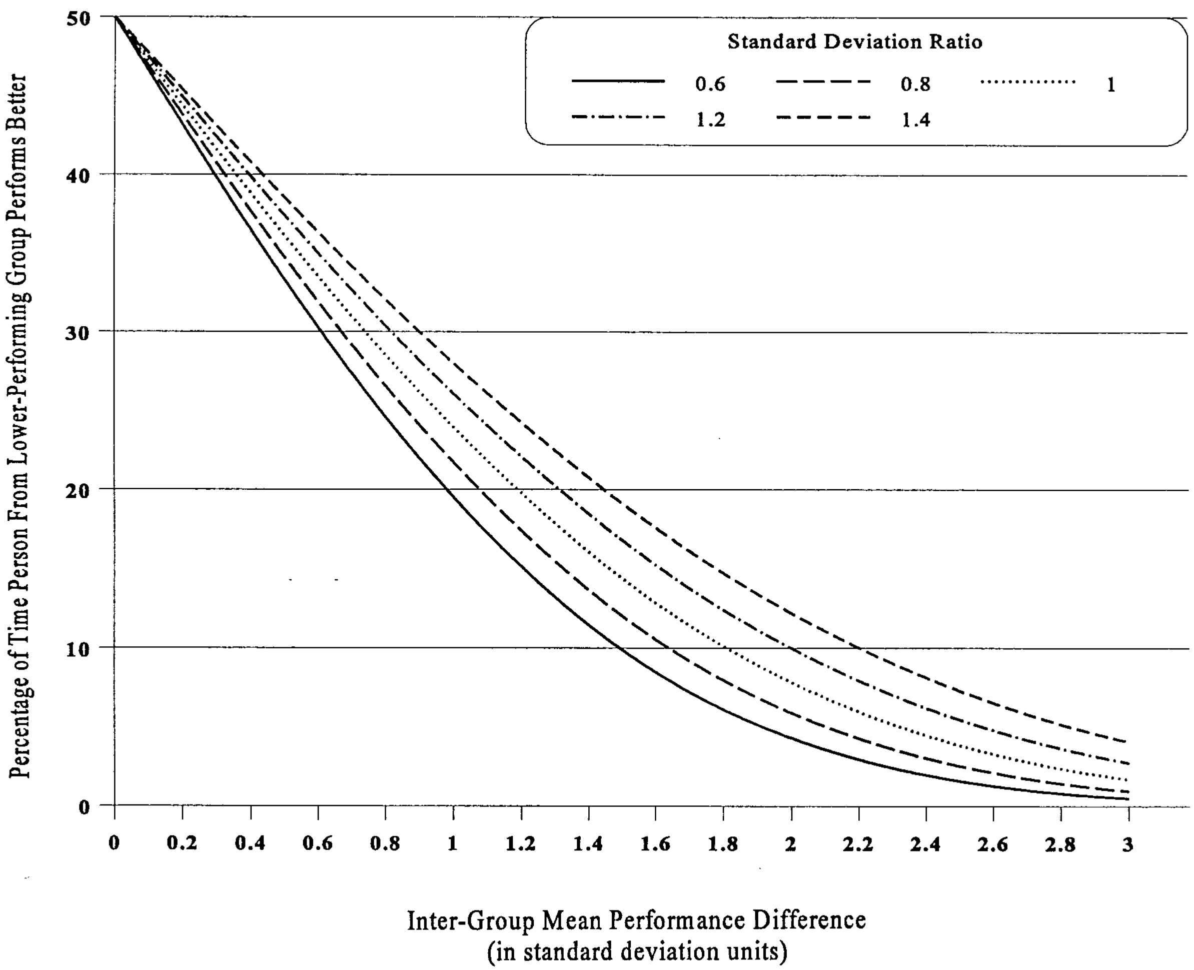
Percent of the Time a Randomly Selected Lower-Performing Group Member is Expected to Outperform a Randomly Selected Higher-Performing Group Member as a Function of Inter-Group Mean Performance Difference When the Standard Deviation Ratio is 1.4[a]

Delta	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.00	50.00	49.77	49.54	49.31	49.08	48.84	48.61	48.38	48.15	47.92
0.10	47.69	47.46	47.22	46.99	46.76	46.53	46.30	46.07	45.84	45.61
0.20	45.38	45.15	44.92	44.69	44.46	44.23	44.00	43.77	43.54	43.31
0.30	43.08	42.85	42.63	42.40	42.17	41.94	41.72	41.49	41.26	41.04
0.40	40.81	40.59	40.36	40.14	39.91	39.69	39.46	39.24	39.02	38.79
0.50	38.57	38.35	38.13	37.91	37.68	37.46	37.24	37.02	36.81	36.59
0.60	36.37	36.15	35.93	35.72	35.50	35.28	35.07	34.85	34.64	34.42
0.70	34.21	34.00	33.78	33.57	33.36	33.15	32.94	32.73	32.52	32.31
0.80	32.10	31.89	31.69	31.48	31.27	31.07	30.86	30.66	30.45	30.25
0.90	30.05	29.85	29.65	29.44	29.24	29.04	28.85	28.65	28.45	28.25
1.00	28.06	27.86	27.67	27.47	27.28	27.09	26.89	26.70	26.51	26.32
1.10	26.13	25.94	25.76	25.57	25.38	25.20	25.01	24.83	24.64	24.46
1.20	24.28	24.10	23.92	23.74	23.56	23.38	23.20	23.02	22.85	22.67
1.30	22.50	22.32	22.15	21.98	21.81	21.64	21.47	21.30	21.13	20.96
1.40	20.79	20.63	20.46	20.30	20.13	19.97	19.81	19.65	19.49	19.33
1.50	19.17	19.01	18.85	18.69	18.54	18.38	18.23	18.08	17.92	17.77
1.60	17.62	17.47	17.32	17.17	17.03	16.88	16.73	16.59	16.44	16.30
1.70	16.16	16.02	15.87	15.73	15.59	15.46	15.32	15.18	15.05	14.91
1.80	14.78	14.64	14.51	14.38	14.25	14.11	13.98	13.86	13.73	13.60
1.90	13.47	13.35	13.22	13.10	12.98	12.85	12.73	12.61	12.49	12.37
2.00	12.25	12.14	12.02	11.90	11.79	11.67	11.56	11.45	11.34	11.22
2.10	11.11	11.00	10.90	10.79	10.68	10.57	10.47	10.36	10.26	10.15
2.20	10.05	9.95	9.85	9.75	9.65	9.55	9.45	9.35	9.26	9.16
2.30	9.07	8.97	8.88	8.78	8.69	8.60	8.51	8.42	8.33	8.24
2.40	8.15	8.07	7.98	7.89	7.81	7.72	7.64	7.56	7.47	7.39
2.50	7.31	7.23	7.15	7.07	6.99	6.92	6.84	6.76	6.69	6.61
2.60	6.54	6.46	6.39	6.32	6.25	6.18	6.11	6.04	5.97	5.90
2.70	5.83	5.76	5.70	5.63	5.56	5.50	5.43	5.37	5.31	5.24
2.80	5.18	5.12	5.06	5.00	4.94	4.88	4.82	4.77	4.71	4.65
2.90	4.59	4.54	4.48	4.43	4.37	4.32	4.27	4.22	4.16	4.11

Notes. Delta represents the inter-group mean performance difference in standard deviation units.
[a] Ratio computed as higher-performing group standard deviation divided by lower-performing group standard deviation.

Figure 1

Percentage of the Time a Randomly Selected Lower-Performing Group Member Is Expected to Outperform a Randomly Selected Higher-Performing Group Member as a Function of Inter-Group Mean Performance Difference and the Ratio of the Standard Deviations of Group Performance



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